Precautionary Saving under Recursive Preferences

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Abstract

Which preference features guide precautionary saving under recursive utility? We investigate this question in a two-period consumption/saving model. We identify two risk-impact channels, marginal expected utility (MEU) in the risk domain and the certainty equivalent (CE) of future consumption. Higher-order preferences are relevant only in the risk domain. Intertemporal preferences just operate in the CE channel. The MEU generally dominates the CE channel. Second-order preferences tend to influence saving more strongly than any higher-order. Saving reactions to return risk are typically negative. Intertemporal risk effects fade out beyond second-order (variance). Wrongly assuming isoelasticity in the risk domain may understate precaution.

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1 Introduction

The relevance of precautionary saving in lifecycle consumption is robustly verified empirically, but the evidence on the preferences motivating it is scant (e.g., Gourinchas and Parker 2001, Attanasio and Weber 2010, Choi et al. 2017, Lugilde et al. 2017). Theoretically established is the importance of higher-order preferences for precautionary choices.[1] For example, in the standard expected utility (EU) framework the sign of the third utility derivative determines whether saving rises in the face of future income risk. Eeckhoudt and Schlesinger (2008) (ES) extend this analysis to increases in higher-order risk on future income or the saving return. Unfortunately, EU equates risk and intertemporal preferences, precluding any investigation of their potential distinct roles. Interestingly, most empirical studies on preferences follow this choice and, typically, focus on only *one* of the three preference aspects that are generally recognized to govern risky intertemporal choices: risk aversion, discounting, and intertemporal substitution. As a result, it is unresolved which preference features actually guide precautionary decisions, and to what extent.

We address these questions starting from a two-period consumption/saving model under recursive utility (RU) as in Kimball and Weil (2009) (KW). By disentangling risk preferences and the elasticity of intertemporal substitution (EIS), RU can represent risk and intertemporal tradeoffs in a very detailed way.[2] We first augment KW's preference conditions for risk-induced saving increases to cover the broad set of risks in ES. We show that risk impacts saving under RU via two channels, each of which hinges on a different set of preferences.

In a second step, we implement the model numerically and investigate magnitudes. The RU parameterization we choose enables independent curvature in relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS), thus generalizing the isoelastic Ep-

[1] See, for example, Drèze and Modigliani (1966/1972), Leland (1968), Sandmo (1970), Rothschild and Stiglitz (1971), Selden (1979), and Kimball (1990). Kimball describes the precautionary saving motive as the "propensity to prepare and forearm oneself" to better cope with future risk.

[2] Many empirical studies suggest that the preferences from the risk and intertemporal domains generally tend to differ. In macroeconomics and finance, for example, RU has helped to resolve the equity-premium and risk-free-rate puzzles (Hall 1988, Weil 1989, Epstein and Zin 1991, Tallarini 2000, Bansal and Yaron 2004, Barro 2009, Binsbergen et al. 2012, Martin 2013, Epstein et al. 2014).

stein and Zin (1989, 1991) and Weil (1990) (EZW) specification. This modeling allows us to account for the non-constant RRA and EIS shapes often observed in domain-specific studies. Moreover, all risk effects we consider relate to Ekern (1980) "increases in N^{th}-degree risk," so that we can isolate the impacts per risk order. In this detailed framework, we study how saving depends on the different preference features in question, and how risk deteriorations of increasing order influence saving.

Our theoretical results motivate the calibrations. Risk impacts saving under RU via marginal EU (MEU) in the risk domain and the certainty equivalent (CE) of future consumption. For the MEU channel, the preference conditions for higher saving resemble the EU case (see ES). In the CE channel, the impact on CE hinges on risk preferences; but the effect of the CE variation depends then on the relative strength of second-order intertemporal and risk preferences. Higher-order preferences are active only in the risk domain, and intertemporal preferences exclusively operate in the CE channel.

Saving reactions to return risk are characterized by the interplay of a positive precautionary and a negative substitution effect. This interplay is instructive regarding the relative importance of higher- and lower-order preferences: for an increase in N^{th}-degree risk under EU the $(N+1)^{th}$ utility derivative carries the precautionary and the N^{th} the substitution effect. While we cannot determine the typical sign of these saving responses analytically, we show that the conditions for saving to increase become the more demanding the higher N. Under RU, these findings apply to the MEU channel.

We compare next the saving impacts from the MEU and CE channels after a risk increase. Interestingly, when the CE channel positively contributes to optimal saving, the total RU precautionary motive is uniformly *smaller* than the precautionary motive from the MEU channel (and *vice versa*). Hence, the two channels do not involve separate precautionary motives which add up to the total motive. This incongruity results from the first-order effect associated with the CE variation: if a CE decrease raises (reduces) future marginal RU, then, less (more) precautionary saving is required to compensate the risk impact.

This comparison of the MEU and CE channels is equivalent to the comparison of saving responses under EU and RU, when EU is identical to RU risk preferences. A question of practical relevance is whether saving responses under intertemporal EU rather reflect RU risk or intertemporal preferences. Theoretical reasoning does not help to decide this matter.

Our first calibration exercise examines how saving depends on various preference features for a given risk. The first finding is that consumption smoothing has a dominating influence on total saving under risk, at the expense of precaution. As a corollary, the effects from altering the EU parameters closely mirror those of changes in RU intertemporal preferences, suggesting that the time-additive EU model primarily conveys information about consumption smoothing, not risk preference. Under RU, most precautionary saving is due to the MEU channel; the CE channel's contribution to saving variations is, in general, minor.

The saving response to return risk is negative in *all* scenarios we consider. The saving-increase conditions are far from being fulfilled, and increasingly so with raising order of the risk deterioration. The mitigation of return risk works such that the risk premium is zero *ex post*, and the precautionary premia are consistently negative. We conclude that the substitution effect typically dominates the precautionary effect.

We finally evaluate the import of departures from constant RRA and EIS shapes. When wrongly assumed, EZW's isoelasticity seems to be the most innocuous if RRA is actually decreasing or EIS is actually increasing. In the reverse cases, saving predictions may diverge significantly. As RRA is typically found increasing, these observations are particularly relevant when analyzing precaution under RU.

The second calibration exercise investigates the magnitude of saving reactions to risk deteriorations of increasing order. We find a meaningful saving response only for second-order risk; third- and higher-order risk effects are effectively moot. These results are robust to all configurations we consider, including increasing the payoff scale of risk to triple the size of background income and setting preferences to extremes. The precautionary fractions in total saving increase with lottery stakes under income risk, but remain similar across stakes under return risk. Higher background income tends to dilute precautionary effects because saving to smooth consumption increases and exogenous risk exposure falls relatively.

In Section 2, we introduce the framework for the theoretical analysis. Section 3 provides the theoretical comparative-statics results and compares the MEU and CE channels. Section 4 explains the analytical framework for the calibration studies in Sections 5 and 6. In Secion 7, we discuss relations to the literature. Section 8 summarizes the main results and concludes.

2 Recursive Utility Model

Consider a decision-maker who receives exogenous income y_t in periods $t = 1, 2$. The first-period income y_1 is split between consumption c_1 and saving s. Any saving earns gross return R in period 2. The decision-maker exhausts all resources in period 2 through consumption c_2. Risk enters from either $\tilde{y}_2$ or $\tilde{R}$, and we mark risky variables with a tilde.[3]

Following KW, the intertemporal utility-maximization problem under RU in form of Kreps and Porteus (1978) preferences is

$$s^* = \underset{s}{\operatorname{argmax}}\, U\left(c_1, \tilde{c}_2\right) = u\left(c_1\right) + \beta u\left(CE(\tilde{c}_2)\right) \quad \text{s.t.} \quad \begin{cases} y_1 = c_1 + s \\ \tilde{y}_2 + s\tilde{R} = \tilde{c}_2 \end{cases} \tag{1}$$

where u is the felicity function, and β is the discount factor associated with the pure rate of time preference. The certainty equivalent of $\tilde{c}_2$,

$$CE(\tilde{c}_2) = \psi^{-1}\left(E_1\left[\psi\left(\tilde{c}_2\right)\right]\right) \tag{2}$$

ranks future consumption according to the risk preference ψ. Thus, ψ addresses the decision-maker's attitude toward risk, while u captures the desire to smooth consumption. One consequence of the Kreps-Porteus axioms is that ψ is a von Neumann-Morgenstern utility function, but u is not. EU appears when $\psi = u$, making it a limiting case of Kreps-Porteus,

[3] Notation containing both $\tilde{y}_2$ and $\tilde{R}$ with a tilde applies under each of the two risk types.

similar as EZW. (We explain the close relation to EZW in Section 5.4.)

According to the first-order condition

$$u'(c_1) = \beta u'(CE(\tilde{c}_2))\, CE'(\tilde{c}_2) \tag{3}$$

the decision-maker saves until the marginal utility from foregoing consumption in period 1 (i.e., saving a marginal amount) is equal to the discounted marginal utility from consuming in period 2 instead.[4] With some abuse of notation for compactness, we denote by CE' the full derivative of the certainty equivalent with respect to s,

$$CE'(\tilde{c}_2) \equiv \frac{dCE(\tilde{c}_2)}{ds} = E_1\left[\frac{\psi'(\tilde{c}_2)}{\psi'(CE(\tilde{c}_2))}\tilde{R}\right] \tag{4}$$

CE' can be interpreted as a "risk-preference adjusted return to saving" (Selden 1979).

Rewriting (3) in form of the Euler condition illustrates the "pricing" interpretation of saving: equilibrium occurs, when the expected discounted net return to saving is nil,

$$\beta\frac{u'(CE(\tilde{c}_2))}{u'(c_1)}CE'(\tilde{c}_2) = 1 \tag{5}$$

The *stochastic discount factor* (SDF) of the saving return involved in (5),

$$\beta\frac{u'(CE(\tilde{c}_2))}{u'(c_1)}\frac{\psi'(\tilde{c}_2)}{\psi'(CE(\tilde{c}_2))}$$

contains all behavioral implications of the model.

The extra terms compared to the EU SDF $\beta\frac{u'_{eu}(\tilde{c}_2)}{u'_{eu}(c_1)}$ reflect the manner in which the CE

[4]Our proofs, which are based on comparative-static exercises at the saving optimum, assume that (3) holds with equality. The exercises also hold at a boundary, albeit rather trivially, so we do not consider boundary issues. The possibility of corner equilibria may be of empirical importance, however (e.g., in the presence of borrowing or lending constraints).

ranking of $\tilde{c}_2$ changes with s. Grouping the u' and ψ' terms in (5) elucidates the difference:

$$\beta \frac{u'\left(CE(\tilde{c}_2)\right)}{u'\left(c_1\right)} = \frac{\psi'\left(CE(\tilde{c}_2)\right)}{E_1[\psi'(\tilde{c}_2)\tilde{R}]}$$

Intuitively, in equilibrium the consumption-smoothing utility tradeoff from saving must equal the risk-preference tradeoff between safe and risky consumption in the future.

To fulfill the second-order condition

$$u''\left(c_1\right) + \beta\left[u''\left(CE(\tilde{c}_2)\right)\left[CE'(\tilde{c}_2)\right]^2 + u'\left(CE(\tilde{c}_2)\right)CE''(\tilde{c}_2)\right] < 0 \tag{6}$$

some restrictions must be placed on u and ψ (KW, Gollier 2001). Namely, in addition to both being increasing and concave, ψ^{-1} must be concave in s. This holds if ψ's absolute risk tolerance – the inverse of absolute risk aversion – is concave. Under these assumptions (which we maintain throughout), $CE''(\tilde{c}_2) < 0$.[5]

Saving has two motivations in this model: consumption smoothing and precaution. The consumption-smoothing part s^{cs} is the amount saved when risk is collapsed to its expectation, $\bar{y}_2 \equiv E_1(\tilde{y}_2)$ or $\bar{R} \equiv E_1(\tilde{R})$. Under RU and EU, s^{cs} is equal,

$$s^{cs} = \underset{s}{\operatorname{argmax}}\ u(y_1 - s) + \beta u(\bar{y}_2 + s\bar{R}) \tag{7}$$

Precautionary saving, then, is the extra amount saved when risk enters, $s^{prec} = s^* - s^{cs}$.

The two motivations rely on different kinds of preferences. Consumption smoothing depends on a decision-maker's relative resistance to intertemporal substitution $RRIS$, equivalent to the inverse of EIS, and is captured by the Arrow-Pratt coefficient $-\frac{u''(c)c}{u'(c)}$. Intuitively, s^{cs} increases with EIS. The precautionary motive depends on higher-order preferences. For example, prudence, or $u'''_{eu} > 0$, guarantees a positive motive in the face of second-order income risk under EU, and the coefficient of relative prudence (RP), $-\frac{u'''_{eu}(c)c}{u''_{eu}(c)}$, measures its

[5]The proof of this result in KW (Appendix A) applies analogously under return risk.

strength for small risks (Kimball).

The precautionary motive under RU is more nuanced. For second-order income risk, the preference conditions for a positive motive coincide with the ones ensuring the second-order condition (6) to hold (KW, Gollier 2001). KW's analog to Kimball's RP coefficient illustrates for small risks how consumption-smoothing and risk preferences are involved:

$$RP_{kw}(c) = RRA_{\psi}(c)\left\{1 + \left[RP_{\psi}(c) - RRA_{\psi}(c)\right] \cdot EIS(c)\right\} \tag{8}$$

RP_{kw} rises with RRA_{ψ}, the elasticity of absolute risk tolerance $RP_{\psi} - RRA_{\psi}$, and u's EIS.

Like ES, we consider below saving reactions to risk increases in form of deteriorations in N^{th}-order stochastic dominance (NSD), and their special case of Ekern increases in N^{th}-degree risk. The latter require beyond NSD that the moments up to $N-1$ be identical, and cover well-known cases: a mean-preserving spread (MPS) (Rothschild and Stiglitz 1970) is an increase in second-degree risk; an increase in downside risk (IDR) (Menezes et al. 1980) is of third degree; and an increase in outer risk (Menezes and Wang 2005) is of fourth degree.

For such increases in large risks, the conditions on RU for precautionary saving are only sufficient. To see why, it is instructive to rewrite the first-order condition (3) using (4):

$$u'(c_1) \;=\; \beta \, \frac{u'(CE(\tilde{c}_2))}{\psi'(CE(\tilde{c}_2))} \, E_1[\psi'(\tilde{c}_2)\tilde{R}] \tag{9}$$

Risk affects future marginal RU on the right-hand side of (9) via two channels, the certainty equivalent of future consumption $CE(\tilde{c}_2)$ and MEU in the risk domain $E_1[\psi'(\tilde{c}_2)\tilde{R}]$. While MEU has a direct positive impact, the influence of a CE variation depends on the interplay of second-order intertemporal and risk preferences (KW): future marginal RU increases (decreases) with CE if and only if, globally,

$$RRIS_u(CE(\tilde{c}_2)) < (>) RRA_{\psi}(CE(\tilde{c}_2)) \tag{10}$$

In the MEU channel, the precautionary motive operates as under EU. But the first-order effect in CE has an ambiguous impact on saving. While CE decreases with risk under risk aversion, whether RRA_{ψ} or $RRIS_u$ is typically higher is an empirical question.[6] Thus, a risk-averse RU decision-maker, who is imprudent in the risk domain, will still increase (decrease) saving in reaction to risk, when future marginal RU increases (decreases) (KW).

The contemporaneous grouping of the terms in (9) also illustrates how preferences and incentives enter the saving problem (Carroll and Kimball 2005, KW). Because saving flows from period 1 to 2, the left- and right-hand sides can, respectively, be thought of as the intrapersonal *supply* and *demand* of saving.[7] Under this reading, risk and risk preferences only affect saving demand, but intertemporal preferences act on both supply and demand. Plots of this supply-and-demand system illustrate our calibrations in Sections 5 and 6.

Depending on the risk type, saving impacts the distribution of final consumption differently. Under income risk, saving exclusively affects mean period-2 consumption, so that the decision-maker can mitigate risk exposure only indirectly. Under return risk, saving magnifies all moments of $\tilde{c}_2$, and its impact is the stronger the higher the moment: the n^{th} moment of $\tilde{c}_2$ is equal to the n^{th} moment of $\tilde{R}$ *times* s^n. Due to the multiplicative character of the saving return, the decision-maker can directly foreclose risk exposure by restricting saving, and this means is the more effective the higher is n. We state these different impacts on risk exposure in Remark 1, which derives by applying the definition of the n^{th} central moment.

Remark 1 (Saving Impact on Exposure to Income Risk or Return Risk) *Denote period-2 consumption under income risk* $\tilde{c}_2^{\tilde{y}_2} \equiv \bar{y} + \tilde{y}_2 + sR$*, and under return risk* $\tilde{c}_2^{R} \equiv \bar{y} + y_2 + s\tilde{R}$*, and let* $m^n(\tilde{z})$ *be the* n^{th} *central moment of random variable* $\tilde{z}$*. Under income*

[6] In frameworks with more than two periods, whether RRA_{ψ} is higher (lower) than $RRIS_u$ relates to the propensity for early (late) risk resolution (e.g., Epstein et al. 2014).

[7] Pursuing this interpretation further, we could invoke the metaphor of "two selves" – a present and a future one – whose mental bargaining determines the equilibrium saving amount. For example, Andersen et al. (2008) refer to a dual-selves model (e.g., Benhabib and Bisin 2005, Fudenberg and Levine 2006) when arguing that responses to immediately rewarded risk tasks are probably temptation-driven, while the responses to tasks with certain but delayed payments are probably self-controlled. We are agnostic regarding the mental mechanisms that underpin these internal tradeoffs. For our purposes, it is sufficient that the equilibrium condition breaks cleanly into current and future incentives.

risk, saving only alters the first moment of $\tilde{c}_2$:

$$m^1(\tilde{c}_2^{y_2}) = \bar{y} + E_1(\tilde{y}_2) + sR \ , \quad m^n(\tilde{c}_2^{y_2}) = m^n(\tilde{y}_2) \quad for \quad n > 2$$

Under return risk, saving magnifies all moments of $\tilde{c}_2$ by the amount s^n:

$$m^1(\tilde{c}_2^{R}) = \bar{y} + y_2 + sE_1(\tilde{R}) \ , \quad m^n(\tilde{c}_2^{R}) = s^n \cdot m^n(\tilde{R}) \quad for \quad n > 2$$

The principally different saving impacts on risk exposure motivate our separate analytical treatment of income risk and return risk, and underpin the different results for the two cases.

3 Comparative Statics

3.1 First-Order Effects

First-order effects directly activate consumption smoothing, and influence precautionary choices indirectly via the saving tradeoff under the budget constraint. The preference features which control a first-order effect differ depending on the parameter inducing it. We consider first-order effects in per-period incomes y_1 and $\tilde{y}_2$, background income $\overline{y}$, and gross saving return $\tilde{R}$. $\overline{y}$ adds an aspect of lifecycle consumption. In the calibrations, this parameter helps us to slide the decision problem to various points of the wealth domain.

Let $F(s;\Theta)$ be the first derivative of objective (1) written with mean incentives $\Theta \equiv \big(y_1, \bar{y}_2, \bar{R}, \overline{y}\big)$. Throughout the paper, we focus on incentives such that optimal total saving is positive.[8] The saving change due to a variation in incentive $\theta \in \Theta$ follows from the implicit function theorem: $\frac{ds^*}{d\theta} = - \frac{\partial F(s;\Theta)/\partial \theta}{\partial F(s;\Theta)/\partial s}\Big|_{s=s^*}$. Because the denominator is equal to condition (6), the numerator alone dictates the sign of the comparative static.

Proposition 1 adapts and completes results on first-order effects under RU.[9]

[8] We make this assumption for convenience and coherence. The results under income risk hold similarly for negative saving amounts. Under return risk, concentrating on positive saving simplifies the exposition.

[9] KW conduct comparative statics on wealth and future income risk, but not on returns. Gollier summa-

Proposition 1 (First-Order Effects on Saving) *Marginal increases in the incentives y_1, $\tilde{y}_2$, $\overline{y}$, and $\tilde{R}$ result in the following directional changes in optimal saving s^*:*

$$\text{sgn}\left[\frac{ds^*}{dy_1}\right] = \text{sgn}\left[-u''(c_1)\right] > 0 \tag{11a}$$

$$\text{sgn}\left[\frac{ds^*}{d\bar{y}_2}\right] = \text{sgn}\left[u''(CE(\tilde{c}_2))\, CE_{y_2}(\tilde{c}_2) CE'(\tilde{c}_2) + u'(CE(\tilde{c}_2))\, CE'_{y_2}(\tilde{c}_2)\right] < 0 \tag{11b}$$

$$\text{sgn}\left[\frac{ds^*}{d\overline{y}}\right] = \text{sgn}\left[-u''(c_1) + \beta\left(u''(CE(\tilde{c}_2))\, CE_{y_2}(\tilde{c}_2) CE'(\tilde{c}_2) + u'(CE(\tilde{c}_2))\, CE'_{y_2}(\tilde{c}_2)\right)\right] \gtreqless 0 \tag{11c}$$

Saving decreases if

- *ψ exhibits DARA and $\frac{u''(CE(\tilde{c}_2))}{u'(CE(\tilde{c}_2))^2}$ is weakly increasing; or*
- *ψ exhibits DARA and decreasing relative prudence, $ARIS_u \geq ARA_\psi$, and $RP_u \leq RP_\psi$.*

$$\text{sgn}\left[\frac{ds^*}{d\bar{R}}\right] = \text{sgn}\left[u''(CE(\tilde{c}_2))\, sCE_{y_2}(\tilde{c}_2) CE'(\tilde{c}_2) + u'(CE(\tilde{c}_2))\left[sCE'_{y_2}(\tilde{c}_2) + CE_{y_2}(\tilde{c}_2)\right]\right] \gtreqless 0 \tag{11d}$$

if and only if
$$-\frac{u''(CE(\tilde{c}_2))}{u'(CE(\tilde{c}_2))} sCE'(\tilde{c}_2) - \frac{CE'_{y_2}(\tilde{c}_2)}{CE_{y_2}(\tilde{c}_2)} s \lesseqgtr 1 \tag{11e}$$

The expressions in (11a)–(11d) arise from straightforward calculation of all derivatives involved.[10] The sign conditions and qualifications in (11c) derive from KW (Appendix A).

The first two effects reflect the preference for smooth consumption: when current income y_1 rises, consumption smoothing involves sending some funds into the future by saving more, and *vice versa* when future income $\bar{y}_2$ rises. The direction of the saving response to a

rizes the corresponding preference conditions under EU. We use the notation:

$$CE_{y_2}(\tilde{c}_2) \equiv \frac{\partial CE(\tilde{c}_2)}{\partial \bar{y}_2} = E_1\psi'(\tilde{c}_2)/\psi'(CE(\tilde{c}_2))$$

$$CE'_{y_2}(\tilde{c}_2) \equiv \frac{\partial CE'(\tilde{c}_2)}{\partial \bar{y}_2} = \left[E_1[\psi''(\tilde{c}_2)\,\tilde{R}] - \psi''(CE(\tilde{c}_2))\, CE_{y_2}(\tilde{c}_2)\, CE'(\tilde{c}_2)\right] / \psi'(CE(\tilde{c}_2))$$

where, with an argument similar to that for $CE'' < 0$, it follows that $CE'_{y_2} < 0$.

[10]For the expression in (11d), note that $\frac{\partial CE(\tilde{c}_2)}{\partial \bar{R}} = sCE_{y_2}(\tilde{c}_2)$ and $\frac{\partial CE'(\tilde{c}_2)}{\partial \bar{R}} = sCE'_{y_2}(\tilde{c}_2) + CE_{y_2}(\tilde{c}_2)$.

y_1 increase depends on the concavity of the intertemporal preference u, whereas, when $\bar{y}_2$ changes, the properties of consumption smoothing and risk aversion ψ apply. An increase in $\overline{y}$ is akin to a permanent income shock. The sign of its effect is ambiguous, hinging on the relative strengths of the effects in (11a) and (11b).

Increasing the future return $\bar{R}$ generates, as under EU, two opposing effects on saving. On the one hand, higher future wealth prompts higher consumption today (i.e., less saving) due to consumption smoothing; the negative terms in (11d) capture this wealth effect. On the other hand, higher future returns reduce the opportunity cost of future consumption by lowering the implicit price of saving; this substitution toward future consumption (i.e., more saving) is captured by the positive term. Saving increases if the substitution effect dominates the wealth effect. This occurs if the left-hand side of (11e) is below 1.

Condition (11e) has a simple interpretation if risk is suppressed. It collapses to:

$$-\frac{u''(\overline{y}+y_2+sR)}{u'(\overline{y}+y_2+sR)}sR \lesseqqgtr 1 \qquad (12)$$

For the case with $\overline{y} = y_2 = 0$ (as in ES), thus, $RRIS_u$ has to be below one (i.e., EIS sufficiently low) for saving to increase. If $ARIS_u$ is decreasing, higher income (e.g., due to $\bar{y} + y_2 > 0$) reduces the left-hand side of (12), and facilitates this condition to be fulfilled.

Conditions (11) show that under RU first-order effects mainly depend on first- and second-order aspects of intertemporal and risk preferences. All effects that involve period 2 simultaneously activate consumption-smoothing and risk preferences. Higher-order preferences only appear when signing the competing effects in the $\overline{y}$ comparative static.

3.2 Income-Risk Effects

We now extend KW's conditions on RU for higher saving in response to income risk to higher-order risk effects. We account for the two risk-impact channels. For the MEU channel, the conditions correspond to the EU case. ES show that an NSD deterioration in $\tilde{y}_2$ makes a decision-maker save more if and only if $\text{sgn}\left[u_{eu}^{(n)}\right] = (-1)^{n+1}$ for $n = 2, \ldots, N+1$. An Ekern increase in N^{th}-degree risk requires a sign condition only for $n = N+1$. For $u_{eu} = \psi$, these conditions apply to the MEU channel. Thus, following a MPS $\psi''' > 0$ guarantees a positive contribution to precautionary saving, and following an IDR $\psi'''' < 0$.

To link a deterioration from $\tilde{y}_{2,l}$ to $\tilde{y}_{2,h}$ to the level of CE, we introduce the risk premium π^{y_2} in the rewritten definition (2):

$$\psi\left(CE(\overline{y} + \tilde{y}_{2,h} + sR)\right) = E_1\psi\left(\overline{y} + \tilde{y}_{2,l} + sR - \pi^{y_2}\right) = E_1\psi\left(\overline{y} + \tilde{y}_{2,h} + sR\right) \tag{13}$$

π^{y_2} captures the decision-maker's maximum willingness to pay to avoid the given risk increase. π^{y_2} increases under NSD deteriorations in $\tilde{y}_2$ if and only if $\text{sgn}\left[\psi^{(n)}(.)\right] = (-1)^{n+1}$ for all $n = 1, 2, \ldots, N$, and under Ekern risk increases if and only if this sign condition holds for $n = N$ (Eeckhoudt and Schlesinger 2006, Li 2009, Denuit and Eeckhoudt 2010).

Distinct preference features are activated per channel in the case of Ekern risk increases. For example, a MPS requires $\psi''' > 0$ for MEU to increase, and $\psi'' < 0$ for CE to decrease; and the constellation of second-order risk and intertemporal preferences as in (10) then decides whether the CE channel contributes positively or negatively to precautionary saving. Similarly, an IDR requires $\psi'''' < 0$ for MEU to increase, and $\psi''' > 0$ for CE to decrease.

By contrast, when considering simultaneous risk increases in at least two adjacent orders, then, given $\psi' > 0$, the conditions on risk preferences for MEU to increase imply the ones for CE to decrease. For example, under a general 2SD deterioration, $\psi', -\psi'', \psi''' > 0$ together guarantee a higher MEU and imply a lower CE. The same implications holds under a general 3SD deterioration when $\psi', -\psi'', \psi''', -\psi'''' > 0$.

Proposition 2 summarizes the conditions for saving increases from NSD deteriorations.

Proposition 2 (Saving Increases for Income-Risk Increases in NSD) *Let $s^*_{y_{2,l}}$ and $s^*_{y_{2,h}}$ be the optimal saving choices from* (3) *under $\tilde{y}_{2,l}$ and $\tilde{y}_{2,h}$, respectively. An NSD deterioration from $\tilde{y}_{2,l}$ to $\tilde{y}_{2,h}$ makes $E_1\psi'(\tilde{c}_2^{y_2})$ increase, and $CE(\tilde{c}_2^{y_2})$ decrease, for all s if and* *only if* $sgn\big[\psi^{(n)}(.)\big] = (-1)^{n+1}$ *for all* $n = 1, 2, \ldots, N+1$*. Then,* $s^*_{y_{2,h}} \geq s^*_{y_{2,l}}$ *if, alternatively,*

- $RRIS_u\left(CE(\tilde{c}^{y_2}_{2,l})\right) \geq RRA_\psi\left(CE(\tilde{c}^{y_2}_{2,l})\right)$ *for all* s*; or*
- $RRIS_u\left(CE(\tilde{c}^{y_2}_{2,l})\right) < RRA_\psi\left(CE(\tilde{c}^{y_2}_{2,l})\right)$ *for all* s*, and the negative effect via* $CE(\tilde{c}^{y_2}_2)$ *on future marginal RU in* (9) *does not overcompensate the positive effect via* $E_1\psi'(\tilde{c}^{y_2}_2)$*.*

Corollary 1 states the preference conditions for saving increases from Ekern risk increases.

Corollary 1 (Saving Increases for Ekern Income-Risk Increases) *Let* $s^*_{y_{2,l}}$ *and* $s^*_{y_{2,h}}$ *be the optimal choices as in Proposition 2. Under an Ekern increase in* N^{th}*-degree risk from* $\tilde{y}_{2,l}$ *to* $\tilde{y}_{2,h}$ *the following two equivalences hold:*

1. $E_1\psi'(\overline{y} + \tilde{y}_{2,h} + sR) \;\geq\; E_1\psi'(\overline{y} + \tilde{y}_{2,l} + sR) \;\; \forall s \;\Leftrightarrow\; \text{sgn}\big[\psi^{(N+1)}(.)\big] = (-1)^N.$
2. $CE(\overline{y} + \tilde{y}_{2,h} + sR) \;\leq\; CE(\overline{y} + \tilde{y}_{2,l} + sR) \;\; \forall s \;\Leftrightarrow\; \text{sgn}\big[\psi^{(N)}(.)\big] = (-1)^{N+1}.$

If 1. and 2. hold as announced, then, $s^*_{y_{2,h}} \geq s^*_{y_{2,l}}$ *follows as in Proposition 2.*

Some interesting comparisons to EU arise. Under RU, higher-order preferences are important only in the risk domain, potentially in both channels. Intertemporal preferences come into play exclusively in the CE channel. To illustrate, consider the response to a MPS. Given $\psi'', -\psi''' < 0$, a RU decision-maker saves more if, in the CE channel, second-order preferences are stronger in the intertemporal than in the risk domain (so that $RRIS_u > RRA_\psi$ in (10)). Should the decision-maker be particularly risk-averse (so that $RRIS_u < RRA_\psi$), then saving still increases if the negative effect from $\psi'' < 0$ does not overcompensate the positive effect from $\psi''' > 0$. For an IDR, similar reasoning applies, only that $\psi''', -\psi'''' > 0$ are required for risk preferences to admit the same effects in the two channels.

3.3 Return-Risk Effects

Under return risk, the preference conditions for the MEU channel correspond, as before, to the EU case. An NSD deterioration on $\tilde{R}$ makes an EU decision-maker save more if, for all

$n = 1, \ldots, N$, the product of the n^{th} derivative of $f'(R) = u'_{eu}(c_2^R)R$ and $(-1)^n$ is positive,

$$(-1)^n \left[u_{eu}^{(n+1)}\left(c_2^R\right) sR + n\, u_{eu}^{(n)}\left(c_2^R\right)\right] \;>\; 0 \tag{14}$$

(ES). Ekern increases in N^{th}-degree risk have (14) for $n = N$ as a necessary and sufficient condition. For clarity, we stick in our following discussion for EU to Ekern risk increases.

Condition (14) has some interesting implications. If $(-1)^{n+1}u_{eu}^{(n)}(.) > 0$ for $n = N, N+1$, the term with $u_{eu}^{(N+1)}$ represents the precautionary and the term with $u_{eu}^{(N)}$ the substitution effect. Thus, a higher N gives more weight to the substitution effect, and makes it outweigh the precautionary effect more easily. Writing (14) as a level condition on the coefficient of partial N^{th}-degree risk aversion illustrates this effect,[11]

$$-\frac{u_{eu}^{(N+1)}(\overline{y} + y_2 + sR)}{u_{eu}^{(N)}(\overline{y} + y_2 + sR)}\, sR \;>\; N \tag{15}$$

The higher is N, the more demanding is this condition. Increasingly, an EU decision-maker will tend to substitute away from future consumption and *decrease* saving.

To link a deterioration from $\tilde{R}_l$ to $\tilde{R}_h$ to the level of CE, we adopt Eeckhoudt and Schlesinger's (2009) multiplicative risk premium. Premium π^R arises by comparing the EU of future consumption as

$$E_1\psi(\overline{y} + y_2 + s(\tilde{R}_l - \pi^R)) = E_1\psi(\overline{y} + y_2 + s\tilde{R}_h) \tag{16}$$

π^R is the proportion of saving such that $s\pi^R$ is equal to the maximum amount of future consumption one is willing to forgo to avoid the given risk increase. π^R increases under the same conditions as π^{y_2} for risk deteriorations of the same degree, so that $\psi'' < 0$ is necessary and sufficient for CE to decrease under a MPS, and $\psi''' > 0$ under an IDR.

[11]Following Menezes and Hanson (1970) and Chiu et al. (2012), we call "coefficient of partial N^{th}-degree risk aversion" a quantity of the form $-\frac{f^{(N)}(x+t)t}{f^{(N-1)}(x+t)}$ for $t > \underline{t}$, where $\underline{t} > 0$ is chosen to make $Pr\,(x + t < 0) = 0$. Equation (12) above states (15) for a *positive* first-order effect on $\tilde{R}$.

Proposition 3 and Corollary 2 extend prior results under RU for MPS, for example by Selden (1979) and Weil (1990), to higher-order risk effects.

Proposition 3 (Saving Increases for Return-Risk Increases in NSD) *Let $s^*_{R_l}$ and $s^*_{R_h}$ be the optimal saving choices from* (3) *under return lotteries $\tilde{R}_l$ and $\tilde{R}_h$. An NSD deterioration from $\tilde{R}_l$ to $\tilde{R}_h$ implies the following two statements:*

1. $E_1[\psi'(\overline{y}+y_2+s\tilde{R}_h)\tilde{R}_h] \;\geq\; E_1[\psi'(\overline{y}+y_2+s\tilde{R}_l)\tilde{R}_l]$ $\forall s$, *if* $-\frac{\psi^{(n+1)}(\overline{y}+y_2+sR)}{\psi^{(n)}(\overline{y}+y_2+sR)}\, sR \;\geq\; n$ *for* $n = 1, 2, \ldots, N$.

2. $CE(\overline{y}+y_2+s\tilde{R}_h) \;\leq\; CE(\overline{y}+y_2+s\tilde{R}_l) \;\; \forall s \quad \Leftrightarrow \quad \mathrm{sgn}\big[\psi^{(n)}(.)\big] = (-1)^{n+1}$ *for* $n = 1, 2, \ldots, N$.

If 1. and 2. hold as announced, then, $s^*_{R_h} \geq s^*_{R_l}$ if, alternatively,

- $RRIS_u\left(CE(\tilde{c}^R_{2,l})\right) \geq RRA_\psi\left(CE(\tilde{c}^R_{2,l})\right)$ *for all s; or*
- $RRIS_u\left(CE(\tilde{c}^R_{2,l})\right) < RRA_\psi\left(CE(\tilde{c}^R_{2,l})\right)$ *for all s, and the negative effect via $CE(\tilde{c}^R_2)$ on future marginal RU* (9) *does not overcompensate the positive effect via $E_1[\psi'(\tilde{c}^R_2)\tilde{R}]$.*

Applying the definition of Ekern risk increases again simplifies the proposition.

Corollary 2 (Saving Increases for Ekern Return-Risk Increases) *Let $s^*_{R_l}$ and $s^*_{R_h}$ be the optimal choices as in Proposition 3. An Ekern increase in N^{th}-degree risk from $\tilde{R}_l$ to $\tilde{R}_h$ implies the following two equivalences:*

1. $E_1[\psi'(\overline{y}+y_2+s\tilde{R}_h)\tilde{R}_h] \;\geq\; E_1[\psi'(\overline{y}+y_2+s\tilde{R}_l)\tilde{R}_l]$ $\forall s \;\Leftrightarrow -\frac{\psi^{(N+1)}(\overline{y}+y_2+sR)}{\psi^{(N)}(\overline{y}+y_2+sR)}\, sR \;\geq\; N$.

2. $CE(\overline{y}+y_2+s\tilde{R}_h) \;\leq\; CE(\overline{y}+y_2+s\tilde{R}_l) \;\; \forall s \quad \Leftrightarrow \quad \mathrm{sgn}\big[\psi^{(N)}(.)\big] = (-1)^{N+1}$.

If 1. and 2. hold as announced, then, $s^*_{R_h} \geq s^*_{R_l}$ follows as in Proposition 3.

RU does not admit simple criteria to predict whether a decision-maker exhibits a positive precautionary motive in reaction to return risk or not. The relevance of our observations regarding the MEU channel in this context depends on the relative importance of the MEU and CE channels in saving responses. We turn to this question in the following.

3.4 Comparing the MEU and CE Channels

How can the MEU and CE channels be compared? Interestingly, this question closely relates to the comparison of saving predictions under RU and EU for the case that EU equates RU risk preferences. The main ingredients for these comparisons are the precautionary premia from Liu (2014) and Bostian and Heinzel (2018).[12]

For EU functions $f \in \{\psi, u_{eu}\}$, the precautionary premia for increases in income risk and return risk, $\theta_f^{y_2}$ and θ_f^R, derive by comparing future marginal EU, respectively, as

$$Ef'(\tilde{y}_{2,l} + sR - \theta_f^{y_2}) = Ef'(\tilde{y}_{2,h} + sR) \tag{17a}$$

$$E[f'(y_2 + s(\tilde{R}_l - \theta_f^R))\tilde{R}_l] = E[f'(y_2 + s\tilde{R}_h)\tilde{R}_h] \tag{17b}$$

$\theta_f^{y_2}$ is the safe reduction in $\tilde{y}_{2,l}$ that has the same effect on saving as the increase from $\tilde{y}_{2,l}$ to $\tilde{y}_{2,h}$. θ_f^R, in turn, is the proportion of saving such that $s\theta_f^R$ is the safe change in $\tilde{c}_2^R$ that has the same effect on saving as the deterioration from $\tilde{R}_l$ to $\tilde{R}_h$. Importantly, θ_f^R is positive or negative depending on whether the precautionary or the substitution effect dominates.

The RU precautionary premia θ^{y_2} and θ^R derive, with analogous interpretations, from

$$u'(CE(\tilde{y}_{2,l} + sR - \theta^{y_2}))CE'(\tilde{y}_{2,l} + sR - \theta_u^{y_2}) = u'(CE(\tilde{y}_{2,h} + sR))CE'(\tilde{y}_{2,h} + sR) \tag{18a}$$

$$u'(CE(y_2 + s(\tilde{R}_l - \theta^R)))CE'(y_2 + s(\tilde{R}_l - \theta_u^R)) = u'(CE(y_2 + s\tilde{R}_h))CE'(y_2 + s\tilde{R}_h) \tag{18b}$$

A direct quantitative link between the preferences that sustain the risk-impact channels and the total precautionary motive arises when inserting the risk premia from (13) and (16)

[12] KW study similar relations in their context comparing RU and EU with reference to compensating premia. We focus on equivalent premia instead.

and the precautionary premia from (17) into equations (18):

$$\frac{u'(CE(\tilde{y}_{2,l}+sR-\pi_{\psi}^{y_2}))}{\psi'(CE(\tilde{y}_{2,l}+sR-\pi_{\psi}^{y_2}))}E\psi'(\tilde{y}_{2,l}+sR-\theta_{\psi}^{y_2}) = u'(CE(\tilde{y}_{2,h}+sR))CE'(\tilde{y}_{2,h}+sR) \quad (19a)$$

$$\frac{u'(CE(y_2+s(\tilde{R}_l-\pi_{\psi}^{R})))}{\psi'(CE(y_2+s(\tilde{R}_l-\pi_{\psi}^{R})))}E[\psi'(y_2+s(\tilde{R}_l-\theta_{\psi}^{R}))\tilde{R}_l] = u'(CE(y_2+s\tilde{R}_h))CE'(y_2+s\tilde{R}_h) \quad (19b)$$

While, by our above assumptions, the left-hand sides of (17) and (18) depend positively on θ_ψ or θ, the effect of π_ψ in (19) is ambiguous: in both cases, θ increases (decreases) with π_ψ when $RRIS_u$ is larger (smaller) than RRA_ψ.

Using these concepts, Proposition 4 compares the saving choices of a RU and an EU decision-maker when EU corresponds to RU risk preferences.

Proposition 4 (Income- and Return-Risk Effects under RU and EU) *Consider a RU and an EU decision-maker with preferences such that $\psi \equiv u_{eu}$, exhibiting DARA.*

1. *Suppose an NSD deterioration in $\tilde{y}_2$ such that both decision-makers save more. Then,*

   (1.i) θ^{y_2} and $\theta_\psi^{y_2}$ are uniformly larger than $\pi_\psi^{y_2}$, and

   (1.ii) if $RRIS_u\left(CE(\tilde{c}_{2,l}^{y_2})\right) \leq (\geq) RRA_\psi\left(CE(\tilde{c}_{2,l}^{y_2})\right)$ for all s, $\theta^{y_2} \geq (\leq)\ \theta_\psi^{y_2}$.

2. *Suppose an NSD deterioration in $\tilde{R}$ such that both decision-makers save less. Then,*

   (2.i) θ^R and θ_ψ^R are uniformly smaller than π_ψ^R, and

   (2.ii) if $RRIS_u\left(CE(\tilde{c}_{2,l}^{R})\right) \leq (\geq) RRA_\psi\left(CE(\tilde{c}_{2,l}^{R})\right)$ for all s, $\theta^R \leq (\geq)\ \theta_\psi^R$.

If both decision-makers save more in response to the NSD deterioration in $\tilde{R}$, then, the comparisons analogous to (2.i) and (2.ii) are, in general, ambiguous.

We prove Proposition 4 in Appendix A.

The proposition admits two interpretations. First, when $u_{eu} = \psi$ and ψ exhibits DARA, an increase in income risk makes a prudent RU decision-maker with a fairly high (low) EIS save at least (at most) as much more as an EU decision-maker. Second, when comparing *within* a RU decision-maker the total precautionary motive θ^{y_2} with the one associated with the MEU channel $\theta^{y_2}_{\psi}$, we see that $\theta^{y_2} \geq (\leq)\ \theta^{y_2}_{\psi}$ if the CE channel contributes negatively (positively) to optimal saving. On its face, this result is surprising because it shows that the MEU and CE channels do not involve separate precautionary motives which add up to the total motive under RU. The first-order effect related to CE explains this phenomenon: for $RRIS_u \leq (\geq) RRA_{\psi}$, the CE decrease reduces (raises) future marginal RU so that more (less) saving is required to compensate the risk impact. The substitution effect due to the CE variation resembles the one associated with $\bar{y}_2$ in (11b), only that condition (10) decides whether $F(s;\Theta)$ decreases or increases with CE.

The two interpretations are similarly valid if saving decreases in response to return risk, only with a reversed prediction for the order of the negative precautionary premia. By contrast, if return risk stimulates higher saving, an order cannot generally be predicted.

This section completes the set of measures of the preferences sustaining each risk-impact channel. The EU precautionary premium θ_{ψ} captures the precautionary motive associated with the MEU channel. The CE channel relies on three measures: π_{ψ} from (13) or (16) for the size of the risk impact on CE, and $RRIS_u$ and RRA_{ψ} in (10) measuring the strength of second-order intertemporal and risk preferences. The difference of the coefficients is decisive for the direction of the saving impact from this channel and also influences its size.

Because the two channels interact, determining their relative saving impacts is not obvious. We can get an idea by considering to what extent each channel contributes to the change in future marginal RU.[13] An additive representation of these contributions derives by equating the log transforms of Euler conditions (5) for the low- and high-risk equilibria,

$$\delta_{MEU} \;\equiv\; \frac{\ln\left(\frac{E_1[\psi'(\tilde{c}_{2,h})\tilde{R}_h]}{E_1[\psi'(\tilde{c}_{2,l})\tilde{R}_l]}\right)}{\ln\left(\frac{u'\left(y_1-s^*_h\right)}{u'\left(y_1-s^*_l\right)}\right)} \;=\; 1-\frac{\ln\left(\frac{u'(CE(\tilde{c}_{2,h}))/\psi'(CE(\tilde{c}_{2,h}))}{u'(CE(\tilde{c}_{2,l}))/\psi'(CE(\tilde{c}_{2,l}))}\right)}{\ln\left(\frac{u'\left(y_1-s^*_h\right)}{u'\left(y_1-s^*_l\right)}\right)} \;\equiv\; 1-\delta_{CE} \tag{20}$$

[13] Because first-order condition (3) is in general not linear in s, the contribution to the change in future marginal RU is no perfect measure of the channel-wise contribution to the saving variation.

For each channel, δ captures the elasticity of the marginal rate of substitution between the high- and the low-risk state of future marginal RU with respect to a percentage change in the marginal rate of substitution between high- and low-risk present consumption.

4 Calibrations

The calibration exercises in Sections 5 and 6 consider decision-makers with a fixed background income $\bar{y}$ in every period, facing no background risk. We analyze their reaction to risk within a saving opportunity which adds to $\bar{y}$. The opportunity involves prospects of the same (expected) level for y_1 and y_2 and a net-positive expected return to saving R. For clarity, we select only scenarios with interior s^*. In other words, $\bar{y}$ is not used for saving. Its role is to move the decision problem to wealth points which seem empirically relevant.

Our selections for u and ψ enable varying shapes of RRA and EIS. This choice reflects two frequent observations from domain-specific studies: an increasing RRA shape from static risk-aversion experiments (e.g., Holt and Laury 2002), and an increasing EIS shape from intertemporal macroeconomic data (e.g., Meyer and Meyer 2005). Isoelasticity also does not fit well revealed third- and fourth-order risk preferences from the Noussair et al. (2014) experiment. While EU is flatly incompatible with the disunion of shapes between the risk and intertemporal domains, the isoelasticity of EZW preferences does just not accommodate the curvature aspect. Our "double expo-power" (DEP) specification uses in each domain an expo-power function $f \in \{\psi, u\}$ (Saha 1993, Xie 2000),

$$f(c) = \frac{1}{\alpha_f}\left[1 - \exp\left(-\alpha_f \cdot \frac{c^{1-\rho_f}}{1-\rho_f}\right)\right] \tag{21}$$

Taken separately, α_f and ρ_f are akin to the "risk-aversion parameters" of exponential (CARA) and power utility (CRRA) functions: f is exponential for $\rho_f = 0$, and it converges to power utility for $\alpha_f \to 0$.[14] The expo-power form melds these two common types,

[14] This expo-power form admits a second power-utility specification if $\rho_f \to 1$ and $\alpha_f > 0$. To eliminate

giving rise to other forms.

To illustrate, consider the two Arrow-Pratt coefficients for ψ:

$$ARA_{\psi}(c) = \alpha_{\psi} c^{-\rho_{\psi}} + \rho_{\psi} c^{-1}\ , \quad RRA_{\psi}(c) = \alpha_{\psi} c^{1-\rho_{\psi}} + \rho_{\psi}$$

When $\alpha_{\psi} > 0$, RRA curvature depends on the level of ρ_{ψ}: RRA increases if $\rho_{\psi} < 1$, and it decreases if $\rho_{\psi} > 1$.[15] Jointly $\alpha_f \to 0$ and $\rho_f \to 1$, moreover, yield log utility, and jointly $\rho_{\psi} = 0$ and $\alpha_{\psi} \to 0$ risk neutrality. By specifying u similarly, we obtain independent flexibility in consumption smoothing: EIS is constant for $\alpha_u \to 0$, and when $\alpha_u > 0$ increasing if $\rho_u > 1$ and decreasing if $\rho_u < 1$.[16]

Our baseline preference calibration is

$$\alpha_{\psi} = 0.03 \qquad \rho_{\psi} = 0.7 \qquad \alpha_u = 0.05 \qquad \rho_u = 1.6 \qquad \beta = 1$$

The ψ parameters follow the estimates in Holt and Laury, while ρ_u falls in the range of many macroeconomic estimates. DRRIS added through a slightly positive α_u. In the calibration exercises, we vary ρ_{ψ} and ρ_u between 0.5 and 2.5, thus addressing a number of possible shapes for RRA and RRIS. We set the discount factor β to 1 to more clearly accentuate the effects of interest here, risk aversion and intertemporal substitution.[17]

The decisions in the calibrations occur relative to a background income of $\bar{y} = \$1,000$, a value on the order of the monthly per-capita income in the first quintile of the US population. The baseline incomes in the saving opportunity are $y_1 = y_2 = \$100$, or 10% of $\bar{y}$. The baseline return R is 1.01, or an annual net rate of 13% in the monthly analogy. The baseline income-

any confusion from duplicate specifications of the same behavior, we disallow this alternative specification.

[15]Excluding pathological parameterizations, the expo-power function has derivatives with alternating sign, concave absolute risk tolerance, and decreasing absolute risk aversion (except in the edge case $\rho_{\psi} = 0$). The underlying optimization problem is thus well-behaved.

[16]Bommier et al. (2016) show that an intertemporal risk response can potentially be rationalized by more than one configuration of KP preferences. This DEP specification thus provides an empirical advantage by accommodating a number of shapes *ex ante*.

[17]Saving demand depends linearly on β, so that the effect on s^* from an increase in the pure rate of time preference is easy to intuit.

lottery risk has 50-50 chances of a \$50 deviation in y_2, and the baseline return-lottery risk has 50-50 chances of a 10 percentage-point deviation in R.

5 Linking Preferences and Risks

We study now numerically the relative importance of different preference features for saving: how do consumption smoothing and precaution interact? How do saving under RU and EU compare? How do the MEU and CE channels impact saving? Are there distinct effects for return risk? What is the import of non-constant RRA and RRIS shapes? Following some graphical illustrations, we approach these questions quantitatively.

5.1 Graphical Analysis

Figure 1 shows the intrapersonal saving market (9) given the baseline income lottery for perturbations of the RU intertemporal and risk preferences, u and ψ, and u_{eu}. The preference variations illustrate how saving depends on the two preference domains and allows to judge which domain intertemporal EU primarily resembles. Moreover, the separate variation of the α and ρ parameters shows their distinct influences under the DEP specification.

In the first two panels, we consider fairly large ranges for α_u and α_ψ, between 0 and 1, vary ρ_u from 0.5 to 1.6, but keep ρ_ψ between 0.7 and 2.5 to better reflect empirical evidence. The most noticeable feature is that the changes in intertemporal preferences generate large shifts in the curves, whereas similar changes in risk preferences imply comparatively small movements (particularly ρ_ψ). The variations in consumption-smoothing preference u affect both saving supply and demand, but in opposite directions; the net effect on total saving is not very large. Risk attitudes ψ enter the model via the certainty equivalent of future consumption (see (1)). ψ addresses exclusively future valuation, and thus only shifts saving demand. Increasing α_ψ or ρ_ψ shifts saving demand out, pointing to increased precautionary

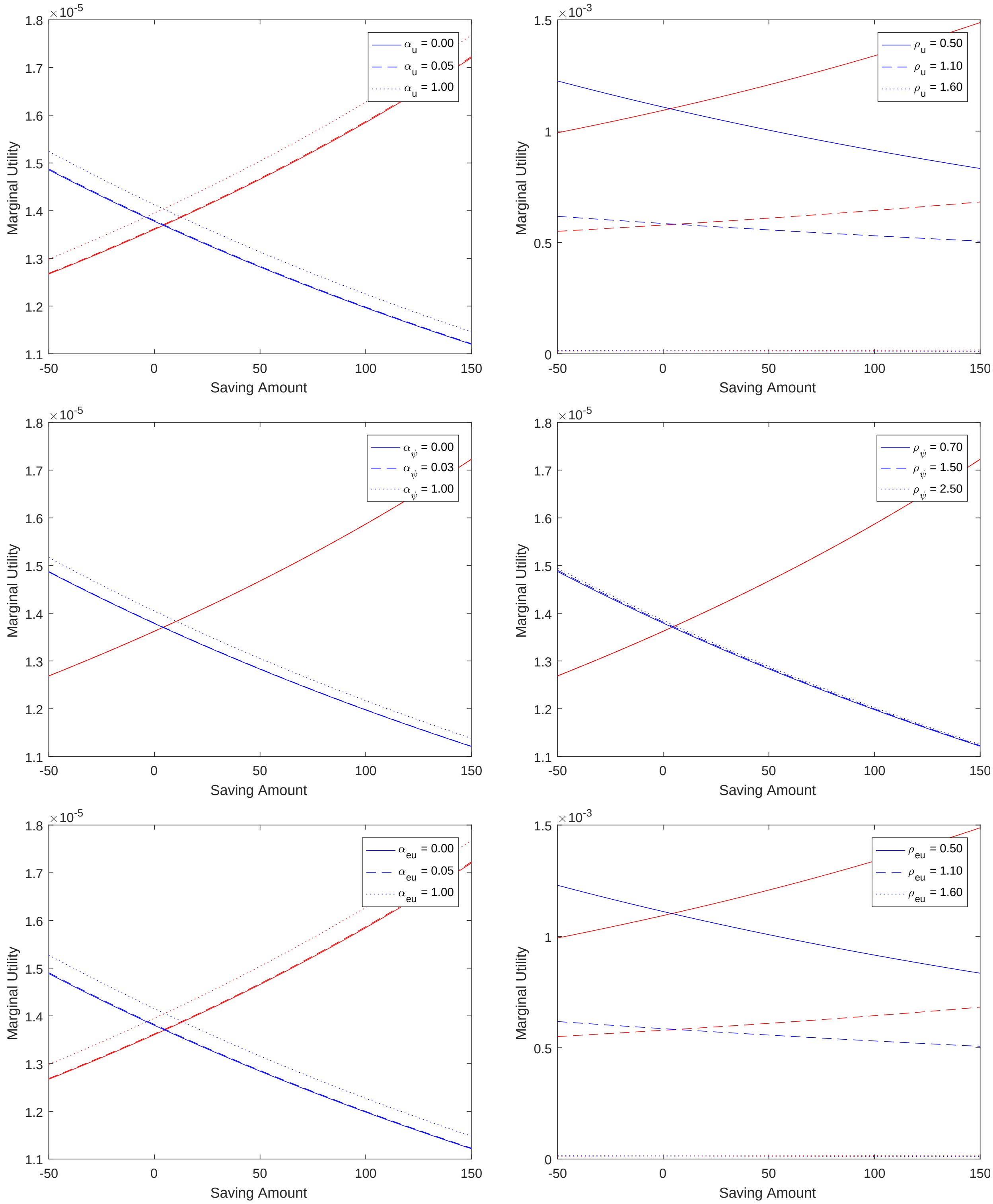


Figure 1: Comparative statics for α_u and ρ_u (top), α_ψ and ρ_ψ (middle), and α_{eu} and ρ_{eu} (bottom) at \$100 scale.

saving. This outcome occurs even though the slope of RRA changes at $\rho_\psi = 1$.[18]

The third panel repeats the graphical analysis with EU preferences whose parameterization follows RU intertemporal preferences, $(\alpha_{eu}, \rho_{eu}) = (\alpha_u, \rho_u)$. (We compare the association of EU with RU risk preferences in Section 5.2.) Under EU, utility curvature is often called "risk aversion." In intertemporal contexts, this term is murky because the same curvature must also explain consumption smoothing. The comparative statics shift both saving supply and demand substantially. These movements do not match the prior shifts in ψ at all, but do bear a marked resemblance to those of u. This result indicates that EU curvature primarily reflects consumption-smoothing preferences.

The different impacts of comparable parameter variations in the two domains illustrate the fundamental importance of intertemporal preferences, which deal with first-order effects, in determining saving under future risk. The second- and higher-order risk effects that risk preferences address appear to be less important.

5.2 Quantitative Comparisons for Income Risk

Table 1 highlights the ρ comparative statics of total saving from Figure 1 quantitatively. As

RU	IRRA (0.03,0.70)	CRRA (0.00,1.50)	DRRA (0.03,2.50)	**EU**
IRRIS (0.05,0.50)	3.28	3.76	4.57	4.08
CRRIS (0.00,1.10)	5.92	6.55	7.61	6.13
DRRIS (0.05,1.60)	4.24	4.77	5.67	4.86
EU	6.81	5.03	4.14	

Table 1: Predictions for total saving under income risk. The values behind the RRA shapes indicate (α_ψ, ρ_ψ), and the values behind the RRIS shapes (α_u, ρ_u) and (α_{eu}, ρ_{eu}).

a reference, we consider constant RRA and RRIS at the middle ρ values, by setting $\alpha = 0$.[19]

In each RRA scenarios, total saving is the highest for CRRIS and the lowest for IRRIS. In every RRIS scenario, increasing ρ_ψ raises total saving, confirming the plots above. When

[18]Optimal saving decreases with ρ_ψ until around 0.75 and increases thereafter.

[19]The scenarios with $\alpha = 0$ yield qualitatively the same results as with the baseline α values. We consider the effects of α variations in Section 5.4.

EU is tied to RU intertemporal preferences, the pattern of EU predictions mimics that of the RU predictions for any given (α_ψ, ρ_ψ), supporting the visual impression that EU resembles RU intertemporal preferences. When the EU parameterization is, instead, tied to RU risk preferences, total saving *decreases* across the three RRA shapes, as opposed to the increasing prediction for any given RRIS shape under RU.

Table 2 decomposes total saving into its consumption-smoothing and precautionary components for a number of scenarios from Table 1. We indicate, in addition, the preference

DRRIS	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	$RRIS_u$	θ^{y_2}	$\pi_\psi^{y_2}$	RRA_ψ	$\theta_\psi^{y_2}$	$\delta_{MEU}^{y_2}$
IRRA	4.24	3.40	0.84	19.74%	1.60	1.69	1.07	0.95	2.11	0.89
CRRA	4.77	3.40	1.37	28.66%	1.60	2.76	1.70	1.50	2.83	0.99
DRRA	5.67	3.40	2.27	40.04%	1.60	4.58	2.82	2.50	3.95	1.13
DRRA										
IRRIS	4.57	2.52	2.05	44.85%	2.16	4.13	2.83	2.50	3.95	1.06
CRRIS	7.61	4.95	2.66	34.94%	1.10	5.37	2.82	2.50	3.94	1.06
DRRIS	5.67	3.40	2.27	40.04%	1.60	4.58	2.82	2.50	3.95	1.13
EU					$RRIS_{eu}$	$\theta_{eu}^{y_2}$		RRA_{eu}	$\theta_{eu}^{y_2}$	
IRRIS	4.08	2.52	1.56	38.20%	2.16	3.14				
CRRIS	6.13	4.95	1.18	19.18%	1.10	2.37				
DRRIS	4.86	3.40	1.46	30.00%	1.60	2.94				
IRRA	6.81	5.76	1.04	47.41%				0.96	2.11	
CRRA	5.03	3.63	1.40	27.85%				1.52	2.83	
DRRA	4.14	2.18	1.96	30.00%				2.53	3.95	

Table 2: Saving predictions and preference measures under income risk (preferences as in Table 1).

measures from Section 3, all at the respective equilibrium. The first block addresses RRA variations given DRRIS. As RRA increases, saving to smooth consumption stays constant, but precautionary saving rises uniformly. The levels of the precautionary premia reflect the latter effect. The MEU channel contributes between 89 and 113% to precautionary saving. When the contribution from the CE channel is positive ($\delta_{CE} = 1 - \delta_{MEU} > 0$), the EU exceeds the RU precautionary premium, and *vice versa*, as predicted by Proposition 4.

The second block addresses RRIS variations given DRRA. Increasing ρ_u from 0.50 does not yield a uniform ordering: saving is the highest at CRRIS. The percentage of saving from precaution correlates negatively with total saving, indicating that total saving variation is mainly driven by consumption smoothing. Interestingly, the RU precautionary premia track the absolute levels of precautionary saving, but not its fractions in total saving. The measures of risk preferences are only indirectly affectedand roughly stay constant.

The EU parameterization in the third block, linked to the u parameter values, generates identical amounts of saving to smooth consumption as under the RRIS variation. Because RU intertemporal preferences have consistently lower ρ values compared to ψ, precautionary saving and the precautionary premia are in each case lower than for the corresponding RRIS scenario under RU. The precautionary saving fractions track their variation under RU.

The EU parameterization in the fourth block follows the ψ parameter values. Precautionary saving increases across the RRA scenarios, as under RU, and the values of the EU precautionary premia match. As Proposition 4 predicts, the EU precautionary saving amounts are higher (lower) than in the corresponding RU cases when $\theta_{eu}^{y_2} > (<)\, \theta^{y_2}$. But, neither the amounts nor the overall pattern of saving to smooth consumption and the precautionary saving fractions resemble the ones from the corresponding RU scenarios.

In Table 2, precaution contributes 19-47% of total saving. Because the expected labor income is constant across periods and $\beta = 1$, saving is mainly due to the positive net return. Eliminating this incentive should restrict saving to precaution, whereas a higher return should reduce the relevance of precaution. Table 3 replicates the saving decomposition for net returns of 0% and 10%, confirming this intuition. Interestingly, between the two cases total saving increases substantially, and precautionary saving even decreases. This provides further evidence that intertemporal preferences drive most of the saving decision.

5.3 Return Risk

The quantitative decomposition for return risk reveals a fundamental difference.[20] Table 4 presents the saving predictions and preference measures for a risky baseline return. Prefer-

[20] The graphical analysis is qualitatively similar to that for income risk, see the Online Appendix.

	$R = 1.0$				$R = 1.1$			
DRRIS	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	s^*	s^{cs}	s^{prec}	s^{prec}/s^*
IRRA	0.85	0.00	0.85	100.00%	31.98	31.22	0.76	2.38%
CRRA	1.38	0.00	1.38	100.00%	32.46	31.22	1.24	3.82%
DRRA	2.30	0.00	2.30	100.00%	33.28	31.22	2.06	6.19%
DRRA								
IRRIS	2.07	0.00	2.07	100.00%	25.04	23.14	1.90	7.58%
CRRIS	2.70	0.00	2.70	100.00%	47.81	45.45	2.36	4.93%
DRRIS	2.30	0.00	2.30	100.00%	33.28	31.22	2.06	6.19%
EU								
IRRIS	1.57	0.00	1.57	100.00%	24.60	23.14	1.45	5.91%
CRRIS	1.19	0.00	1.19	100.00%	46.49	45.45	1.04	2.24%
DRRIS	1.47	0.00	1.47	100.00%	32.55	31.22	1.32	4.07%

Table 3: Saving predictions under income risk for $R = 1.0, 1.1$ (preferences as in Table 1).

ences and other incentives are as in Table 2. A striking result is that precautionary saving is consistently *negative*. That is, the same prudent decision-makers who increase saving in response to income risk save *less* when facing return risk. Given the direct endogeneity of risk exposure, they prefer to foreclose risk by reducing saving. The risk premia in the new equilibrium are zero, and the precautionary premia are negative.

The channel-wise contributions to the saving variation (20) are to be interpreted with caution. Here, the proportions from the MEU channel quantify the contributions to the saving decrease. Via the CE channel, saving *increases* to the extent that δ^R_{MEU} is larger than one. When $\delta^R_{MEU} < 1$, the CE channel also contributes to the saving decrease. The first-order effect from the CE channel may be so strong that δ^R_{MEU} may even be negative.

When u_{eu} is associated with the u parameter values, the EU saving patterns are again close to those of the RU intertemporal domain. But for $u_{eu} = \psi$, no resemblance with a RU preference domain is apparent. Interestingly, the precautionary saving fraction in total saving is similar across all preference scenarios, and the multplicative precautionary premia consistently match with those of the MEU channel. We do not indicate preference intensity coefficients because of their specific form under return risk (Bostian and Heinzel 2018).

Remark 1 and conditions (12) and (15) help to get some intuition as to why saving tends

DRRIS	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	$RRIS_u$	θ^R	π^R_ψ	RRA_ψ	θ^R_ψ	δ^R_{MEU}
IRRA	3.39	3.40	-0.010	-0.29%	1.60	-0.006	0.00	0.95	-0.010	1.41
CRRA	3.39	3.40	-0.016	-0.46%	1.60	-0.009	0.00	1.50	-0.010	1.06
DRRA	3.38	3.40	-0.026	-0.76%	1.60	-0.015	0.00	2.50	-0.010	0.43
DRRA										
IRRIS	2.51	2.52	-0.014	-0.57%	2.16	-0.011	0.00	2.50	-0.010	0.84
CRRIS	4.90	4.95	-0.054	-1.11%	1.10	-0.022	0.00	2.50	-0.010	-0.28
DRRIS	3.38	3.40	-0.026	-0.76%	1.60	-0.015	0.00	2.50	-0.010	0.43
EU						θ^R_{eu}			θ^R_{eu}	
IRRIS	2.51	2.52	-0.012	-0.50%		-0.010				
CRRIS	4.93	4.95	-0.024	-0.49%		-0.010				
DRRIS	3.39	3.40	-0.017	-0.49%		-0.010				
IRRA	5.73	5.76	-0.028	-0.49%					-0.010	
CRRA	3.61	3.63	-0.018	-0.49%					-0.010	
DRRA	2.17	2.18	-0.011	-0.49%					-0.010	

Table 4: Saving predictions and preference measures under return risk (preferences as in Table 1).

to decrease. Remark 1 shows that saving directly magnifies the n^{th} moment of $\tilde{R}$ by s^n. For example, the variance of $\tilde{R}$ is weighted by s^2. Reducing saving is thus a very effective means to reduce risk exposure. The level conditions (15) for saving to increase become the tighter the higher is N. Finally, the left-hand sides of (12) and (15) decrease with income $\overline{y} + y_2$, if absolute $(N+1)^{th}$-degree risk aversion is decreasing for the appropriate $N \in \mathbb{N}$.

Table 5 complements Table 4 by indicating the saving-increase criteria for a first-order deterioration, a MPS, and an IDR of $\tilde{R}$ for the MEU channel and EU. The entries show the values of the partial risk-aversion coefficients (12) for the first order, and (15) for $N = 2, 3$, at the optimum minus the respective level requirement. An entry should be positive for saving to increase. All entries, however, are negative, and the further away from zero the higher is the order of the risk deterioration. The values increase only marginally when evaluating at a hypothetical zero background income, despite decreasing coefficient shapes.

Additional comparative statics in Section 6, for increased risk and varied preferences (e.g., strong risk aversion, $\alpha_\psi = 0.5$), confirm this picture. We conclude that in the saving reaction to return risk the substitution effect typically outweighs the precautionary effect.

	$\bar{y} = \$1,000$			$\bar{y} = \$0$		
RU	$-\frac{\psi''(c_2^R)}{\psi'(c_2^R)}sR$ -1	$-\frac{\psi'''(c_2^R)}{\psi''(c_2^R)}sR$ -2	$-\frac{\psi''''(c_2^R)}{\psi'''(c_2^R)}sR$ -3	$-\frac{\psi''(c_2^R)}{\psi'(c_2^R)}sR$ -1	$-\frac{\psi'''(c_2^R)}{\psi''(c_2^R)}sR$ -2	$-\frac{\psi''''(c_2^R)}{\psi'''(c_2^R)}sR$ -3
DRRIS						
IRRA	-0.997	-1.994	-2.991	-0.997	-1.994	-2.991
CRRA	-0.995	-1.992	-2.989	-0.995	-1.992	-2.989
DRRA	-0.992	-1.989	-2.986	-0.992	-1.989	-2.986
DRRA						
IRRIS	-0.994	-1.992	-2.990	-0.988	-1.983	-2.978
CRRIS	-0.989	-1.984	-2.980	-0.989	-1.984	-2.980
DRRIS	-0.992	-1.989	-2.986	-0.992	-1.989	-2.986
EU	$-\frac{u''(c_2^R)R}{u'(c_2^R)}sR$ -1	$-\frac{u'''(c_2^R)}{u''(c_2^R)}sR$ -2	$-\frac{u''''(c_2^R)}{u'''(c_2^R)}sR$ -3	$-\frac{u''(c_2^R)}{u'(c_2^R)}sR$ -1	$-\frac{u'''(c_2^R)}{u''(c_2^R)}sR$ -2	$-\frac{u''''(c_2^R)}{u'''(c_2^R)}sR$ -3
IRRIS	-0.995	-1.994	-2.992	-0.995	-1.991	-2.987
CRRIS	-0.995	-1.991	-2.986	-0.995	-1.991	-2.986
DRRIS	-0.995	-1.992	-2.989	-0.995	-1.992	-2.989

Table 5: Saving-increase critera from (12) and (15) for a first-order deterioration, MPS, and IDR of $\tilde{R}$ for background income at baseline (left) and zero (right) (preferences as in Table 1). An entry should exceed 0 for saving to increase.

5.4 Constant Versus Non-Constant RRA and RRIS Shapes

We finally highlight the incremental effect of moving to non-constant RRA or RRIS shapes. The link between the ubiquitous EZW specification, with its constant RRA and RRIS, and our Kreps-Porteus framework arises when considering an infinite-horizon version of model (1) with two power functions $u(c) = c^{\rho_u}$ and $\psi(c) = c^{\rho_\psi}$. The utility objective thus reads:

$$U_t = u^{-1}\left((1-\bar{\beta})u(c) + \bar{\beta}u(CE(U_{t+1}))\right) = \left[(1-\bar{\beta})c_t^{\rho_u} + \bar{\beta}\left[E_t\left(U_{t+1}^{\rho_\psi}\right)\right]^{\frac{\rho_u}{\rho_\psi}}\right]^{\frac{1}{\rho_u}} \tag{22}$$

The EIS of U_t is $(1-\rho_u)^{-1}$, and its RRA is $1-\rho_\psi$. To obtain a two-period formulation as in (1), consider a finite-horizon decision of this sort from the perspective of the next-to-last period t. Because $t+1$ is the final period, the future utility-generating quantity U_{t+1} is just c_{t+1}. Given that, the outer u^{-1} of U_t is a positive monotone transformation of an additive, non-recursive function, and so it can be discarded from the maximization problem. Finally, by comparing Euler conditions, the discount factor can be translated by setting $\beta = \frac{\bar{\beta}}{1-\bar{\beta}}$.

EZW's constant EIS and RRA fix prudence rather trivially: inserting for EIS and RRA in the KW prudence coefficient (8) yields the constant precautionary motive

$$RP_{EZW} = (1 - \rho_\psi) \left[1 + (1 - \rho_u)^{-1}\right]$$

This prediction cannot account for heterogeneity in u or ψ across wealth levels.

We restate in Table 6 the RU parts of Tables 2 and 4 for CRRIS and CRRA, adding in italics cases equivalent to EZW. (We suppress coefficient curvature again by setting $\alpha = 0$.)

Income Risk

CRRIS	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	$RRIS_u$	θ^{y_2}	$\pi_\psi^{y_2}$	RRA_ψ	$\theta_\psi^{y_2}$	$\delta_{MEU}^{y_2}$
IRRA	5.92	4.95	0.97	16.43%	1.10	1.96	1.07	0.95	2.11	0.99
CRRA	*5.70*	*4.95*	*0.75*	*13.13%*	*1.10*	*1.51*	*0.79*	*0.70*	*1.92*	*0.98*
CRRA	6.55	4.95	1.60	24.44%	1.10	3.23	1.69	1.50	2.82	1.02
DRRA	7.61	4.95	2.66	34.94%	1.10	5.37	2.82	2.50	3.94	1.06
CRRA	*7.61*	*4.95*	*2.66*	*34.94%*	*1.10*	*5.37*	*2.82*	*2.50*	*3.94*	*1.06*
CRRA										
IRRIS	3.76	2.52	1.23	32.85%	2.16	2.48	1.70	1.50	2.83	0.89
CRRIS	*13.67*	*10.89*	*2.48*	*18.57%*	*0.50*	*5.04*	*1.68*	*1.50*	*2.81*	*0.35*
CRRIS	6.55	4.95	1.60	24.44%	1.10	3.23	1.69	1.50	2.82	1.02
DRRIS	4.77	3.40	1.37	28.66%	1.60	2.76	1.70	1.50	2.83	0.99
CRRIS	*4.77*	*3.40*	*1.37*	*28.66%*	*1.60*	*2.76*	*1.70*	*1.50*	*2.83*	*0.99*

Return Risk

CRRIS	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	$RRIS_u$	θ^R	π_ψ^R	RRA_ψ	θ_ψ^R	δ_{MEU}^R
IRRA	4.93	4.95	-0.024	-0.42%	1.10	-0.008	0.00	0.95	-0.01	1.14
CRRA	*4.94*	*4.95*	*-0.015*	*-0.31%*	*1.10*	*-0.006*	*0.00*	*0.70*	*-0.01*	*1.37*
CRRA	4.92	4.95	-0.033	-0.67%	1.10	-0.013	0.00	1.50	-0.01	0.63
DRRA	4.90	4.95	-0.054	-1.11%	1.10	-0.022	0.00	2.50	-0.01	-0.28
CRRA	*4.90*	*4.95*	*-0.054*	*-1.11%*	*1.10*	*-0.022*	*0.00*	*2.50*	*-0.01*	*-0.28*
CRRA										
IRRIS	2.51	2.52	-0.009	-0.34%	2.16	-0.007	0.00	1.50	-0.01	1.31
CRRIS	*10.74*	*10.89*	*-0.156*	*-1.45%*	*0.50*	*-0.029*	*0.00*	*1.50*	*-0.01*	*-0.99*
CRRIS	4.92	4.95	-0.033	-0.67%	1.10	-0.013	0.00	1.50	-0.01	0.63
DRRIS	3.39	3.40	-0.016	-0.46%	1.60	-0.009	0.00	1.50	-0.01	1.06
CRRIS	*3.39*	*3.40*	*-0.016*	*-0.46%*	*1.60*	*-0.009*	*0.00*	*1.50*	*-0.01*	*1.06*

Table 6: Saving predictions and preference measures under non-constant and constant RRA or RRIS shapes and income risk or return risk (preferences as in Table 1 unless differently indicated).

Several important observations arise. First, for decreasing coefficients, saving behavior tends to approximate the corresponding isoelastic case. By contrast, for increasing coefficients, departures may be substantive: while in the risk domain restricted to precaution, in the intertemporal domain both saving types increase.[21]

These results are relevant from the standard EZW perspective. The empirical import of the departures in the intertemporal domain is probably low; decreasing EIS or a constant EIS below one are rarely observed. But, in the risk domain IRRA is a usual case, and departures may be critical precisely when analyzing precautionary behaviors.

6 Risk Effects

How do risk increases of increasing order impact saving? We consider, in turn, mean-preserving spreads and higher-order risk effects for various incentives and preferences.

6.1 Mean-Preserving Spreads

Income Risk

Risk increases affect future marginal utility and so the demand side on the intrapersonal saving market (9). Figure 2 plots this market for MPS of income risk at 1x and 5x lottery scales. To generate MPS of the baseline income lottery (1x scale), we fix the mean income at \$100 and the probabilities at 50-50, and vary the standard deviation from \$0 to \$75 by adding and subtracting equally-sized amounts to each outcome.[22] For MPS at 5x scale, we multiply the baseline MPS prospects by 5, so that the variances increase accordingly. Given (else) the baseline parameter values from Section 4, saving increases with second-order income risk. But the size of the saving response critically depends on lottery stakes: at a 5x scale the saving responses are substantially higher than at the baseline level. In the first case, the

[21] For the intertemporal domain, the difference is somewhat overstated, as the effects increase with α, and we have set $\alpha_u > \alpha_\phi$. It is still clear that RRIS curvature has a relatively higher impact on s^{cs} than s^{prec}.

[22] When constructing MPS, we stop increasing the variance before any lottery outcome becomes negative.

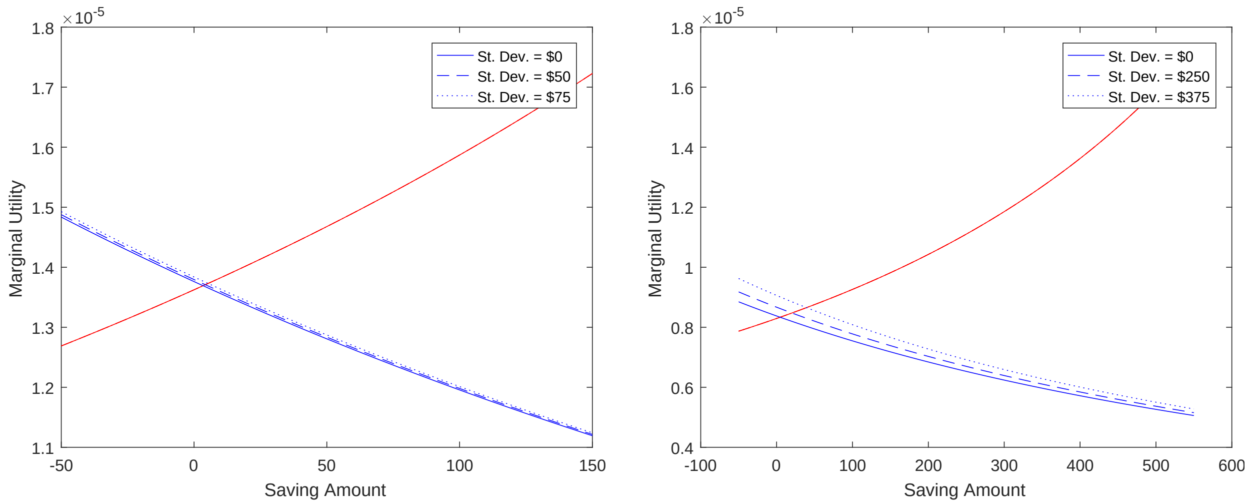


Figure 2: Comparative statics for MPS of $\tilde{y}_2$ lottery at \$100 and \$500 scales.

lottery represents 50% of background income $\bar{y}$ and in the latter 10%.

Breaking saving down into its components, Table 7 shows that the saving increase in response to MPS purely results from precaution. With lottery scaling, saving for consump-

1x Lottery Scale					**5x Lottery Scale**				
RU	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	**RU**	s^*	s^{cs}	s^{prec}	s^{prec}/s^*
0	3.40	3.40	0.00	0.00%	0	4.64	4.64	0.00	0.00%
50	4.24	3.40	0.84	19.74%	250	22.23	4.64	15.59	77.06%
75	5.28	3.40	1.88	35.61%	375	39.40	4.64	34.76	88.22%
EU					**EU**				
0	3.40	3.40	0.00	0.00%	0	4.64	4.64	0.00	0.00%
50	4.86	3.40	1.46	30.01%	250	30.75	4.64	26.11	84.91%
75	6.68	3.40	3.27	40.04%	375	61.68	4.64	57.04	92.48%

Table 7: Impact of $\tilde{y}_2$ MPS on saving under RU and EU at 1x and 5x lottery scales.

tion smoothing increases to some extent. But the higher scaling also gives more discretion for precautionary saving: its predicted proportions rise from about 20% and 36% for the medium- and highest-stakes lottery at the baseline, to 77% and 88% at the 5x scale. Under EU, the effects are analogous, but precautionary saving is even higher.

In the Online Appendix, we consider two variants on these comparative statics. First, tripling background income to $\bar{y} = \$3,000$ dilutes the lottery stakes and considerably attenuates saving even at the highest scaling. Saving to smooth consumption is much higher in both absolute and relative terms, and the precautionary saving fractions fall. Under EU, the attenuation is similar. The second variant considers extreme preferences. At $\alpha_\psi = 0.5$, the saving response is much stronger due to substantially higher precautionary saving, whereas reducing ρ_u to 0.5 decreases the EIS, and with it the saving to smooth consumption. Under EU, the effects of reducing ρ_{EU} to 0.5 correspond to those from reducing ρ_u. But increasing α_{EU} to 0.5 leads to a small drop in s^{cs} and increase in s^{prec} – much like the effect of changing α_u under RU, highlighting the fact that EU primarily reflects intertemporal preference.

Return Risk

The return-risk exercises confirm the main intuition from income risk, with some twists. Figure 3 shows the intrapersonal saving market for MPS of return risk, involving 0, 25, and 75 percentage-point deviations from $R = 1.01$. The lotteries involve thus monthly net interest

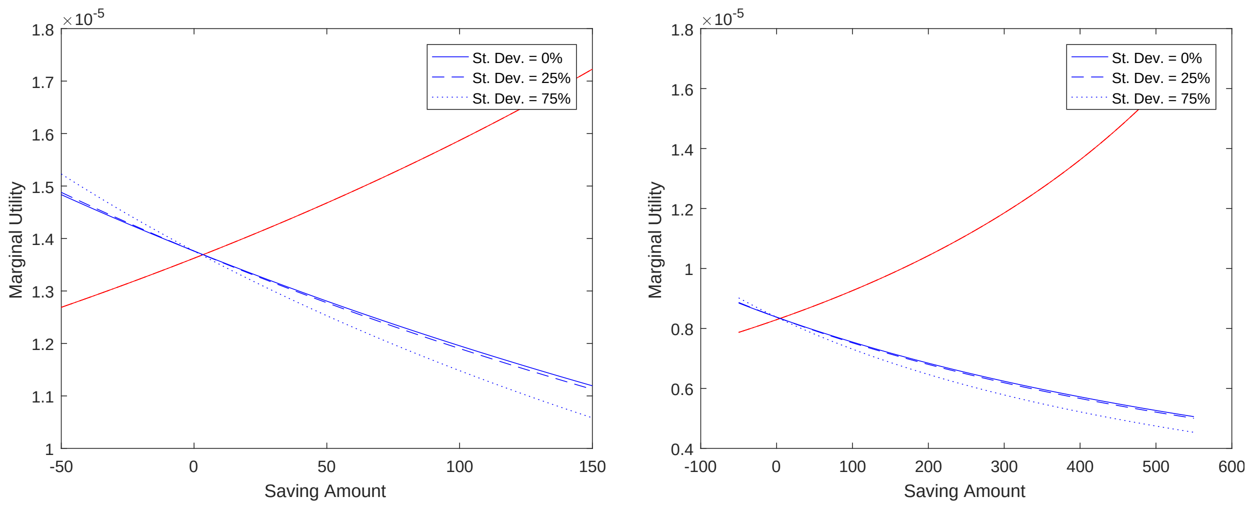


Figure 3: Comparative statics for MPS of R lottery at \$100 and \$500 scales.

rates from -24% to 26%, and -74% to 76%. As above, the substitution and precautionary effects balance so that no positive precautionary saving arises. Saving does not respond very strongly to risk changes. Here, the exogenous risk imposed does not vary across scales, and the shifts in saving demand do not change nearly as much as under income risk.

Breaking saving down into its components in Table 8 shows identical saving for consumption smoothing as under income risk (Table 7), but total saving is consistently lower due to the negative saving response. Interestingly, the tempering of risk exposure operates at

1x Lottery Scale					**5x Lottery Scale**				
RU	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	**RU**	s^*	s^{cs}	s^{prec}	s^{prec}/s^*
0	3.40	3.40	0.00	0.00%	0	4.64	4.64	0.00	0.00%
25	3.34	3.40	-0.06	-1.81%	25	4.55	4.64	-0.08	-1.85%
75	2.93	3.40	-0.48	-16.28%	75	3.98	4.64	-0.66	-16.69%
EU					**EU**				
0	3.40	3.40	0.00	0.00%	0	4.64	4.64	0.00	0.00%
25	3.30	3.40	-0.10	-3.06%	25	4.50	4.64	-0.14	-3.06%
75	2.67	3.40	-0.74	-27.58%	75	3.64	4.64	-1.00	-27.58%

Table 8: Impact of $\tilde{R}$ MPS on saving under RU and EU at 1x and 5x lottery scales.

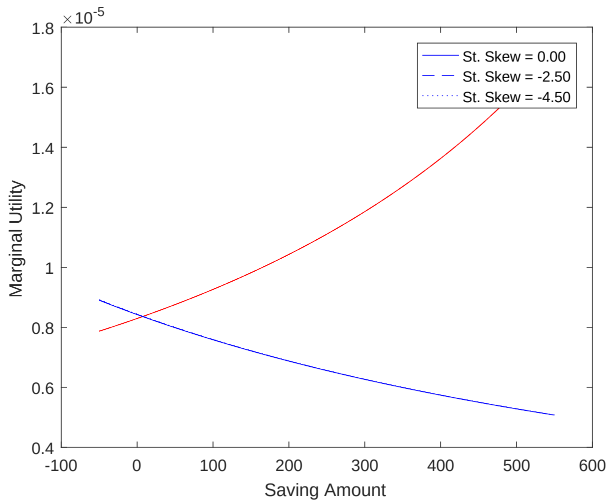
similar proportions across scales. Under EU, outcomes are similar.

6.2 Higher-Order Risk Effects

How do risk increases of higher orders affect saving? Figure 4 illustrates the intrapersonal saving market under increases in downside risk (IDR) of the baseline $\tilde{y}_2$ and $\tilde{R}$ lotteries. To generate an IDR, we induce left skew while preserving the mean and variance of the original lottery (e.g., Eeckhoudt and Schlesinger 2006). For income risk, we start with a (varied) income MPS of $\pm\$20$, and subtract amounts from both outcomes while transferring probability mass from the lower to the upper.[23] The lower outcome moves left to a greater extent than the upper outcome, and this shift combined with the probability transfer gives the lower outcome the appearance of a left-tail outlier. We vary the standardized skew from 0 to about -4.5, and scale the IDR lotteries from up to 30x. For return risk, we similarly construct a IDR starting from a MPS of ± 1 percentage point around $R = 1.01$.

The graphs in Figure 4 show the reactions to IDRs at the 5x scale. Interestingly, the shifts remain indistinguishable to the eye. This vanishing impact of third-order risk is no small-stakes effect: at the 5x scale, the lottery prospects equal half of background income $\bar{y}$.

[23] Constructing an IDR involves a tradeoff between variance and skewness. We reduce the variance here as compared to the baseline in order to allow for greater skewness increases.



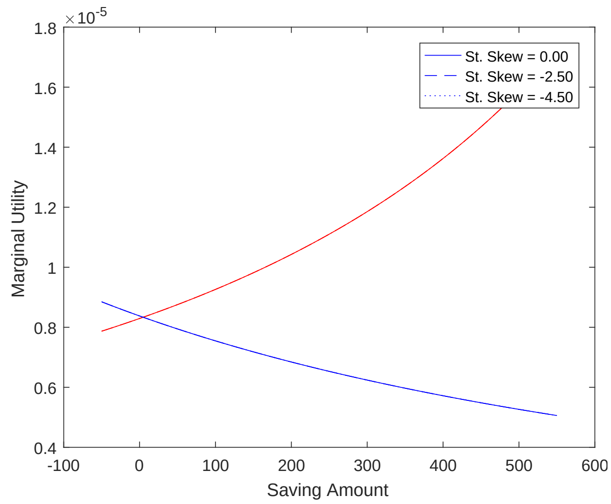


Figure 4: Comparative statics for IDR of (varied) $\tilde{y}_2$ lottery (left) and $\tilde{R}$ lottery (right) at \$500 scale.

Table 9 confirms that the numerical results are consistent with Proposition 2. In interpreting these numbers, it is important to remember that each IDR necessarily includes

$\tilde{y}_2$ **IDR**					$\tilde{R}$ **IDR**				
RU	s^*	s^{cs}	s^{prec}	s^{prec}/s^*	**RU**	s^*	s^{cs}	s^{prec}	s^{prec}/s^*
0	7.15	4.64	2.51	35.10%	0	4.64	4.64	-0.0001	-0.0030%
-2.5	7.58	4.64	2.94	38.76%	-2.5	4.64	4.64	-0.0001	-0.0030%
-4.5	8.05	4.64	3.41	42.34%	-4.5	4.64	4.64	-0.0001	-0.0030%
EU					**EU**				
0	8.91	4.64	4.27	47.92%	0	4.64	4.64	-0.0002	-0.0049%
-2.5	9.95	4.64	5.31	53.36%	-2.5	4.64	4.64	-0.0002	-0.0049%
-4.5	11.20	4.64	6.56	58.56%	-4.5	4.64	4.64	-0.0002	-0.0049%

Table 9: Impact of $\tilde{y}_2$ IDR and $\tilde{R}$ IDR on saving under RU and EU at 5x lottery scale.

second-order risk (positive variance) that also induces precautionary saving. When this second-order effect is removed, the increases in precautionary saving that are solely attributable to the new third-order income risk are about \$1.90 (7.24%) of total saving under RU, and \$2.29 (10.64%) under EU, that is, respectively, about 17.10% and 18.17% of the precautionary saving fractions. Under return risk, no IDR effect of any practical relevance is detectable, under RU and EU. The consumption-smoothing and precautionary reactions continue to balance in such a way that the final impacts on saving demand are minute.

The cases with 10x and 30x lottery scales in the Online Appendix show no essential difference. Because at 30x scale precautionary saving due to second-order risk is already high, the IDR effects are lower than at 10x scale. Under income risk, for $\alpha_\psi = 0.5$, which induced a large risk response for MPS, effects become graphically visible at 10x scale.

Thus, both income and return IDR appear to yield negligible risk responses in both preference models. Given the tiny third-order results, we are unlikely to find meaningful saving responses to fourth-degree and higher risks, and we do not present any further analysis at higher orders. Such effects would be marginal to fourth-order preferences. Our results suggest that third-order preferences may be the largest that can be reasonably observed in practice, at least in the presence of large second-order risks.

7 Relation to Literature

Our analysis of the preference features determining precautionary saving under RU contributes to several strands of literature, which deal with precautionary saving, RU, higher-order risk effects, and measuring the related preferences. The literature crossing precaution with any of the three latter topics is surprisingly thin. Our basic motivation for this paper comes from the work directly aimed at measuring the precautionary saving motive (e.g., Dynan 1993, Merrigan and Normandin 1996, Eisenhauer 2000, Ventura and Eisenhauer 2006, Lee and Sawada 2007, Sawada et al. 2011, Fagereng et al. 2017). All of these studies focus on EU frameworks and start from field observations, and all but one exclusively refer to second-order risk on exogenous income.[24] Overall, their results on the prevalence and strength of a precautionary saving motive remain inconclusive. In part, the estimation methodologies have strongly been criticized (Carroll 2001, Ludvigson and Paxson 2001, Feigenbaum 2005). Our main concern here is with the underlying structural model.

Within the theoretical literature on precautionary saving, only few studies consider RU

[24] Fagereng et al. account for different risk sources, including shocks to consumption needs, non-firm related human capital and the firm, which passes them partially through to workers, as well as partial self-insurance.

(e.g., Selden 1979, Weil 1990, 1993, Langlais 1995, KW, Nocetti and Smith 2010, 2011a,b, Bommier et al. 2012). Despite ES's seminal work on higher-order risk effects on saving under EU, all of these papers focus on second-order risk. KW has been the only constructive contribution to the measurement and comparison of precautionary motives under RU. Bostian and Heinzel (2018) extend this analysis to higher-order risk effects and return risk.

Among the large applied literature using recursive preferences, we are aware of only two studies with a full focus on precautionary saving. Both use the EZW framework and consider second-order income risk. Wang et al. (2016) quantify in a macroeconomic calibration exercise many of the relations and effects studied in the theoretical literature. Using empirical data for the two countries, Choi et al. (2017) attribute a minimum of 50% of saving in China and the U.S. to precaution, but find that the high saving rate in China, despite the much higher income risk, is particularly due to the elevated income growth compared to the U.S.

In macroeconomics and finance, a number of papers investigate the impacts of higher-order risk, including stochastic volatility and rare events, often in an EZW framework (e.g., Weitzman 2007, Barro 2009, Caldara et al. 2012, Martin 2013, Gollier 2016). However, this literature typically considers simultaneous risk changes at the first *and* higher orders, and does not account for the distinction of additive and multiplicative risks or precaution.

Higher-order risk effects have also been the subject of experiments eliciting risk preferences in static settings, as motivated by Eeckhoudt and Schlesinger (2006). The authors observe statistically significant results for reactions to risk manipulations up to the fourth order, entailing fourth-order utility effects (Deck and Schlesinger 2010, 2014, Ebert and Wiesen 2011, 2014, Noussair et al. 2014).

Finally, applied papers studying time-related choices with utility curvature imputed from static risk-aversion experiments have, in varying degree, difficulty to interpret their EU-based results (e.g., Schechter 2007, Andersen et al. 2008, Johansson-Stenman 2010).

Our results contribute to this prior literature in several ways. First, the analysis of higher-order risk effects on precautionary saving clarifies that responses to risk under RU involve higher-order preferences just in the risk domain and intertemporal preferences only until the second order. This structuring shows how experiments on risk preferences on the one hand and research on consumption smoothing on the other can be important for quantitative explorations of precautionary saving. In addition, the findings on non-constant coefficient shapes (Section 4) make a more flexible modeling of RU empirically desirable.

Second, our finding that first-order effects tend to have a stronger effect on saving than any risk effect explains Choi et al.'s observation that a major part of the saving gap between China and the U.S. is due to the elevated income growth in China, and not precaution.

Third, we complement the literature on higher-order risk impacts in macroeconomics and finance by an analysis specific per risk order, and find in tendency declining impacts with increasing risk order. Our explicit distinction between additive and multiplicative risks, moreover, points to fundamentally different behavioral patterns for the two risk types.

Fourth, our calibrations extend the quantitative analysis of higher-order risk effects to intertemporal decisions. For reactions to future risk, we predict insignificance already for the third risk, and thus fourth utility, order. This result rationalizes Deck and Schlesinger's (2014) failure to detect fifth- and sixth-order effects in static experimental tasks, but is even more limiting as in intertemporal choice N^{th}-order risk induces $(N+1)^{th}$-order utility effects.

Fifth, the result that the time-additive EU model primarily conveys information about intertemporal preference calls for caution when using the utility parameters obtained from static risk-aversion experiments for predictions with intertemporal EU. Ongoing discussions on an appropriate experimental design to elicit utility discount rates further reflect this issue (e.g., Andersen et al. 2014, Andreoni et al. 2015).

8 Conclusion

We investigate analytically and by way of calibrations which preference features guide precautionary saving under RU. Our main findings are:

1. Risk impacts saving under RU via two channels, marginal EU (MEU) in the risk domain and the certainty equivalent (CE) of future consumption. The main precautionary response is due to the MEU channel.

2. In precautionary responses, higher-order preferences are relevant only in the risk domain. Intertemporal preferences exclusively operate in the CE channel.

3. Quantitatively, consumption smoothing tends to have a dominating influence on total saving under risk, at the expense of precautionary saving.

4. Risk at third and higher orders is not likely to be meaningful for saving.

5. In response to return risk, saving typically decreases.

6. The time-additive EU model primarily conveys information about intertemporal preference, not risk preference.

7. Wrongly assuming isoelasticity tends to be inconsequential in the intertemporal domain, but in the risk domain may lead to understate precaution.

Our distinction of two risk-impact channels on saving under RU (Result 1) clarifies KW's finding that a decision-maker who is imprudent in the MEU channel may still have a positive – or negative – precautionary response, namely via the CE channel. At the same time, our numerics show that the MEU channel generally dominates precautionary saving responses, so that this surprising case in KW may empirically not be very relevant. For precautionary responses, higher-order preferences are essential only in the risk domain (Result 2). Intertemporal preferences als have their role but are probably of minor importance.

A crucial finding is that consumption smoothing, and thus first-order effects, have an overwhelming influence on total saving, limiting the scope for precautionary saving (Result 3). The numerics show that this effect may be fairly strong. It explains Choi et al.'s result that the elevated saving in China as compared to the U.S. is particularly due to high economic growth and not high risk. By distinguishing saving effects per risk order, we obtain that most precautionary saving is due to second-order risk (positive variance), while the saving effects of third- and higher-order risk (skewness, kurtosis, etc.) are vanishing (Result 4).

We find fundamentally different reaction patterns for risks that are additive *versus* multiplicative to the optimal choice: while income risk tends to increase precautionary saving, the saving response to return risk is typically negative (Result 5). Under return risk, direct risk mitigation by substituting away from the risk (substitution effect) is generally favored over indirect mitigation by saving more (precautionary effect). Remark 1 further shows that risk mitigation via saving reduction is the more effective the higher the risk order. Pursuing the scope and implications of these results seems important in view of the wide analytical importance of multiplicative risks, for example, regarding portfolio investments, production under price or output risks, or labor/leisure decisions under wage-rate risk.

The interpretation of EU curvature in intertemporal choice has been an issue of perennial confusion. Result 6 shows that associating utility curvature purely with risk aversion, as the univariate intertemporal EU model suggests, is misleading regarding the main preference feature at work, namely consumption smoothing. However, fully identifying utility curvature instead with consumption-smoothing preference would exclude the main feature at the basis of risk effects, namely higher-order risk preferences. These results reflect the importance of disentangling risk and intertemporal preferences for a more detailed view of the preferences sustaining precautionary saving.

We also show how RU can rationalize in a single framework the different RRA and EIS shapes typically found in domain-specific studies under EU. We study the import of isoelasticity as associated with EZW preferences, if wrongly assumed. While inconsequential in the intertemporal domain, when assumed in the risk domain precaution may be understated (Result 7). Moving beyond isoelasticity in the risk domain seems thus important for an accurate analysis of precautionary saving. This might also help to avoid seemingly absurd RRA values often found in the applied literature (e.g., Chen et al. 2013).

Our theoretical and quantitative conjectures await further empirical verification and scrutiny in more general settings. Throughout, we consider one risk at a time and focus on saving as the only risk-response instrument. In actuality, several risks and endogenous factors co-determine a decision-maker's risk exposure. For example, the risk position may also depend on labor/leisure, portfolio, and insurance choices, as well as self-protection and self-insurance decisions for specific risks (Flodén 2006, Heinzel and Peter 2016). The two-period framework is robust enough to capture the main tradeoffs over time. Integrating more flexible preferences, return risk, and higher-order risk effects in multi-period analyses, as in Wang et al., could both refine our results and enrich such studies.

For a more detailed knowledge of the preferences sustaining precaution, it seems important to better approach the diversity of preferences. In this paper, we concentrate on comparisons of RU and the standard time-additive univariate EU model. Other approaches to intertemporal choice could potentially prove yet more flexible. For example, Bommier (2007) and Andersen et al. (2018) consider a multivariate utility function over time-dated outcomes, for which they define RRA, the EIS, and correlation aversion. The latter attitude reflects a preference over dated lotteries for those mixing good and bad outcomes, rather than those with only good or only bad outcomes. Their approach avoids the betweenness axiom (Chew 1989, Dekel 1986) implicit in RU and does not involve an explicit utility discount rate. Andersen et al. emphasize that the multivariate utility modeling is amenable to other non-recursive EU specifications, such as rank-dependent utility and prospect theory. We are not aware of any other applied study using this modified-EU approach. Its predictions for precaution and higher-order risk in intertemporal choice remain to be elaborated.

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Appendix

A Proof of Proposition 4

Income-risk increases. In (1.i), $\theta_\psi^{y_2} \geq \pi_\psi^{y_2}$ follows from DARA of ψ. Starting from (13), we have for all $\delta \geq 0$

$$E\psi(\tilde{y}_{2,l} + sR - \pi_\psi^{y_2} + \delta) \leq E\psi(\tilde{y}_{2,h} + sR + \delta)$$

so that at the margin, for $\delta = 0$,

$$E\psi'(\tilde{y}_{2,l} + sR - \pi_\psi^{y_2}) \leq E\psi'(\tilde{y}_{2,h} + sR) \tag{23}$$

Comparison with (17a) along with $\psi'' \leq 0$ implies the result. $\theta^{y_2} \geq \pi^{y_2}$ is then due to (19a), because, with $CE'' < 0$, the left-hand side of (18a) depends positively on θ^{y_2}.

In (1.ii), we have $\theta^{y_2} \geq (\leq)\ \theta_\psi^{y_2}$ from (17a) and $\theta_\psi^{y_2} \geq \pi_\psi^{y_2}$, given the indicated condition:

$$\begin{aligned}\frac{u'(E\psi(\tilde{y}_{2,l} + sR - \theta_\psi^{y_2}))}{\psi'(E\psi(\tilde{y}_{2,l} + sR - \theta_\psi^{y_2}))} E\psi'(\tilde{y}_{2,l} + sR - \theta_\psi^{y_2}) &= \frac{u'(E\psi(\tilde{y}_{2,l} + sR - \theta_\psi^{y_2}))}{\psi'(E\psi(\tilde{y}_{2,l} + sR - \theta_\psi^{y_2}))} E\psi'(\tilde{y}_{2,h} + sR) \\ &\leq (\geq) \frac{u'(E\psi(\tilde{y}_{2,l} + sR - \pi_\psi^{y_2}))}{\psi'(E\psi(\tilde{y}_{2,l} + sR - \pi_\psi^{y_2}))} E\psi'(\tilde{y}_{2,h} + sR)\end{aligned}$$

Due to (13), this is equal to (18a), whose positive dependence on θ^{y_2} implies the result.

Return-risk increases. (2.i) follows trivially when the decision-makers decrease saving, as $\theta^R, \theta_\psi^R \leq 0$ and for any risk-averter $\pi_\psi^R \geq 0$. In (2.ii), $\theta^R \leq (\geq)\ \theta_\psi^R$ is due to similar arguments as under income risk. If both decision-makers increase saving, the analog of (23) has no obvious comparison with (17b). ■